Use Separate Answer scripts for each group

Undergraduate Examination 2019

Semester – III (CBCS)

Mathematics

Generic Elective Course: GEC-3

(Differential Equations and its applications)

Time: Three Hours | Full Marks: 60

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.

<u>Group-A (Marks: 40)</u>

Answer ***any four*** questions. 10×4=40

1. a) Determine the differential equation, whose primitive is $x\cos\theta + y\sin\theta = 5$, where θ is a parameter. 3

   b) Solve the following differential equations:

   i) $\left(x^2+1\right)\dfrac{dy}{dx} - 2xy = x\left(x^4+2x^2+1\right)\cos x,$

   ii) $\left(x^2y^3+2xy\right)dy = dx.$ 3+4

2. a) Show that $\left\{x\left(x^2-y^2\right)\right\}^{-1}$ is an integrating factor of the equation $\left(x^2+y^2\right)dx - 2xy\,dy = 0.$ 3

   b) Solve the following differential equations:

   i) $\dfrac{dy}{dx} = \sin\left(x+y\right),$

   ii) $\left(y^2+2x^2y\right)dx + \left(2x^3-xy\right)dy = 0.$ 3+4

3. a) Find the equation of the family of curves, that cut a system of concentric circles $x^2+y^2=a^2$ at an angle 45°. 3

   b) Find the orthogonal trajectories of the following families of curves:

   i) $r\left(1+\cos\theta\right) = 2a,$

   ii) $x^2+y^2+2\lambda x+9=0,$ where λ is a parameter. 4+3

4. a) Solve the differential equation $y\left\{x\left(2x+1\right)p - yp^2\right\} = 2x^3,\ p = \dfrac{dy}{dx}.$ 3

   b) Solve $y = px + \sqrt{a^2p^2+b^2}$ and obtain the singular solution, if any. 4

   c) Reduce the differential equation $y = 2px + y^2p^3$ by substituting $y^2 = v$ to Clairaut's form and then solve it. 3

5. a) Show that linearly independent solutions of $y'' - 2y' + 2y = 0$ are $e^x\sin x$ and $e^x\cos x$. What is the general solution? Find the particular solution $y\left(x\right)$ with the conditions $y\left(0\right)=1$ and $y'\left(0\right)=-3.$ 4

   b) i) Solve $\dfrac{d^2y}{dx^2} - 9\dfrac{dy}{dx} + 20y = x^2e^{3x}.$

   ii) Solve $\dfrac{d^2y}{dx^2} + 4y = \sin 2x.$ 3+3

P.T.O.

6. a) Solve $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=\sin\left(\log_e x^2\right)$. 3

b) Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2}+y=\cos x$. 4

c) Solve the equations $\frac{dx}{dt}=-wy$ and $\frac{dy}{dt}=wx$ and show that the point (x,y) lies on a circle. 3

Group-B (Marks: 20)

Answer ***any two*** questions. 10×2=20

7. a) Form a partial differential equation (PDE) by eliminating the arbitrary function ϕ from $z=e^{ax+by}\phi(ax-by),\ z=z(x,y)$. 3

b) Give one example of each of the following exclusive types of first order PDE in two independent variables:
i) linear ii) semilinear iii) quasilinear iv) nonlinear. 2

c) Find the general solution of the PDE $\frac{\partial z}{\partial x}+2019\frac{\partial z}{\partial y}=2020$. Hence, show that the Cauchy problem $\frac{\partial z}{\partial x}+2019\frac{\partial z}{\partial y}=2020,\ z(x,y)=2020$ on the line $2019x-y=0$ has no solution. 3+2

8. a) Find the equation of the integral surface of the PDE

$$2y(z-3)\frac{\partial z}{\partial x}+(2x-z)\frac{\partial z}{\partial y}=y(2x-3),$$

which passes through the circle $z=0,\ x^2+y^2=2x$. 5

b) Using Charpit's method, find a complete integral of $x^2\left(\frac{\partial z}{\partial x}\right)^2+y^2\left(\frac{\partial z}{\partial y}\right)^2=4$. 5

9. a) Show that the Partial differential equations $x\frac{\partial z}{\partial x}=y\frac{\partial z}{\partial y}$ and $z\left(x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}\right)=2xy$ are compatible and find the one-parameter family of common solutions. 2+3

b) What do you mean by a singular integral as applied to the solution of first order nonlinear PDE $f\left(x,y,z,\frac{\partial z}{\partial x},\frac{\partial z}{\partial y}\right)$?

Hence, find the singular integral of $z=x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}+3\left(\frac{\partial z}{\partial x}\cdot\frac{\partial z}{\partial y}\right)^{1/3}$. 1+4

———